Research on Student Learning

Research suggests using word problems as a basis for teaching addition and subtraction concepts, rather than teaching computational skills first and then applying them to solve problems. [1] Research has identified a developmental progression of concepts and skills that students use for addition and subtraction. [2] There is evidence that instruction based on a developmental progression of concepts and skills that students use for addition and subtraction can help. [3]

Students make a variety of errors in multi-digit addition and subtraction calculations. [4] Students errors suggest students interpret and treat multi-digit numbers as single digit numbers placed adjacent to each other, rather than using place-value meanings for the digits in different positions. [5] Research also suggests students interpret multiplication of whole numbers mainly as repeated addition. This interpretation is inadequate for many multiplication problems and can lead to restrictive intuitive notions such as "multiplication always makes larger." [6]

Elementary- and middle-school students make several errors when they operate on decimals and fractions. [7] For example, many middle-school students cannot add 4 + 0.3 correctly or 7 1/6 + 3 1/2. [8] Also, lower middle-school students may have difficulties understanding the relationship between fractions and decimals. [9] These errors are due in part to the fact that students lack essential concepts about decimals and fractions and have memorized procedures that they apply incorrectly. Interventions to improve concept knowledge can lead to increased ability by 5th-graders to add and subtract decimals correctly. [10]

Students of all ages misunderstand multiplication and division. [11] Commonly held misconceptions include "multiplication always makes larger" and "division always makes smaller." Students may correctly select multiplication as the operation needed to calculate the cost of gasoline when the amount and unit cost are integers, then select division for the same problem when the amount and unit cost are decimal numbers. [12] Suggestions have been made to improve students concepts of multiplication. [13]

The use of calculators in K-12 mathematics does not hinder the development of basic computation skills and frequently improves concept development and paper-and-pencil skills, both in basic operations and in problem solving. [14] The use of calculators in testing produces higher scores than paper-and-pencil efforts in problem solving as well as in basic operations. [15]

Good estimators use a variety of estimating tactics and switch easily between them. They understand place value and the meaning of operations, and are skilled in mental computation. Poor estimators rely on algorithms that are more likely to yield the exact answer. They lack an understanding of the notion and value of estimation and often describe it as "guessing". [16] Before 6th grade, students develop few estimation skills from their natural experiences. [17] Some researchers caution that teaching estimation to young children may have as its single effect that they master specific procedures in a superficial manner. [18]

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