Research on Student Learning

During preschool and elementary-school years, children develop meanings for number words in which sequence, count, and cardinal meanings of number words become increasingly integrated. [1] Students' own meanings for number words determine to some extent their strategies for adding and subtracting and the complexity of problems they can solve.
Elementary- and middle-school students may have limited ability with place value. [2] Sowder reports that middle-school students are able to identify the place values of the digits that appear in a number, but they cannot use the knowledge confidently in context (for example, students have trouble determining how many boxes of 100 candy bars could be packed from 48,638 candy bars). [3]
Upper elementary- and middle-school students often do not understand that decimal fractions represent concrete objects that can be measured by units, tenths of units, hundredths of units, and so on. [4] For example, students have trouble writing decimals for shaded parts of rectangular regions divided into 10 or 100 equal parts. [5] Other students have little understanding of the value represented by each of the digits of a decimal number or know the value of the number is the sum of the value of its digits. Students of all ages have problems choosing the largest or the smallest in a set of decimals with different numbers of digits to the right of the decimal points. [6] Upper elementary-school students can establish rich meanings for decimal symbols and do a variety of decimal tasks well after specially designed instruction using base-10 blocks. [7]
Upper elementary- and middle-school students may exhibit limited understanding of the meaning of fractional number. [8] For example, many 7th-graders do not recognize that 5 1/4 is the same as 5 + 1/4. [9] In addition, elementary-school students may have difficulties perceiving a fraction as a single quantity. [10] An intuitive basis for developing the concept of fractional number is provided by partitioning [11] An intuitive basis for developing the concept of fractional number is provided by seeing fractions as multiples of basic units -- for example, 3/4 is 1/4 and 1/4 and 1/4 rather than 3 of 4 parts. [12]
Middle-school and even high-school students may have limited understanding about the nature and purpose of estimation. They often think it is inferior to exact computation and equate it with guessing. [13] Middle-school and even high-school students often do not believe estimation is useful. [14] Students who see estimation as a valuable tactic for obtaining information use estimation more frequently and successfully. [15]
There is very little research into student understanding of number symbols as arbitrary conventions. It does indicate that not until 11 years of age do most children consider that correct counting with nonstandard symbols is as adequate as correct counting with standard symbols. [16]

References

[1] Fuson, K., Richards, J., Briars, D. (1982). The acquisition and elaboration of the number word sequence. In Brainerd, C. (Ed.), *Progress in cognitive development research, Vol. 1: Children's logical and mathematical cognition* (pp. 33-92).

Fuson, K. (1988). *Children's counting and concepts of number*.

[2] Sowder, J. (1992). Making sense of numbers in school mathematics. In Leinhardt, G., R. Putnam, and R. Hattrup (Ed.), *Analysis of arithmatic for mathematics teaching* (pp. 1-51).

[3] American Association for the Advancement of Science, Project 2061 (2007). *Atlas for Science Literacy*, 2, 4.

[4] Hiebert, J. (1992). Mathematical, cognitive, and instructional analyses of decimal fractions. In Leinhardt, G., R. Putnam, and R. Hattrup (Ed.), *Analysis of arithmetic for mathematics teaching* (pp. 283-322).

[5] Hiebert, J., Wearne, D. (1986). Procedures over concepts: The acquisition of decimal number knowledge. In Hiebert, J. (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 199-223).

[6] Carpenter, T., Corbitt, M., Kepner, H., Lindquist, M., Reys, R. (1981). Decimals: Results and implications from the second NAEP mathematics assessment. *Arithmetic Teacher*, 28, 34-37.

Hiebert, J., Wearne, D. (1986). Procedures over concepts: The acquisition of decimal

number knowledge. In Hiebert, J. (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 199-223).

Resnick, L., Nesher, P., Leonard, F., Magone, M., Omanson, S., Peled, I. (1989). Conceptual bases of arithmetic errors: The case of decimal fractions. *Journal for Research in Mathematics Education*, 20, 8-27.

[7] Wearne, D., Hiebert, J. (1988). Constructing and using meaning for mathematical symbols: The case of decimal fractions. In Hiebert, J. and M. Behr (Ed.), *Number concepts and operations in the middle grades* (pp. 220-235).

Wearne, D., Hiebert, J. (1989). Cognitive changes during conceptually based instruction of decimal fractions. *Journal of Educational Psychology*, 81, 507-513.

[8] Kieren, C. (1992). The learning and teaching of school algebra. In Grouws, D. (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 390-419).

[9] Kouba, V., Brown, C., Carpenter, T., Lindquist, M., Silver, E., Swafford, J. (1988). Results of the fourth NAEP assessment of mathematics: Numbers, operations, and word problems. *Arithmetic Teacher*, 35, 14-19.

[10] Sowder, J. (1988). Mental computation and number comparison: Their roles in the development of number sense and computational estimation. In Hiebert, J. and M. Behr (Ed.), *Number concepts and operations in the middle grades* (pp. 182-197).

[11] Kieren, C. (1992). The learning and teaching of school algebra. In Grouws, D. (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 390-419).

[12] Behr, M., Lesh, R., Post, T., Silver, E. (1983). Rational number concepts. In Lesh, R. and M. Lindau (Ed.), *Acquisition of mathematical concepts and processes* (pp. 91-126).

[13] Sowder, J. (1992). Estimation and number sense. In Grouws, D. (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 371-389).

[14] Sowder, J., Wheeler, M. (1989). The development of concepts and strategies used in computational estimation. *Journal for Research in Mathematics Education*, 20, 130-146.

[15] Threadgill-Sowder, J. (1984). Computational estimation procedures of young children. *Journal of Educational Research*, 77, 332-336.

[16] Saxe, G., Becker, J., Sadeghpout, M. (1989). Developmental differences in children's understanding of number word conventions. *Journal for Research in Mathematics Education*, 20, 468-488.