

### Research on Student Learning

Upper elementary- and middle-school students may exhibit limited understanding of the meaning of fractional number. [1] For example, many 7th graders do not recognize that  $5 \frac{1}{4}$  is the same as  $5 + \frac{1}{4}$ . [2] In addition, elementary-school students may have difficulties perceiving a fraction as a single quantity. [3] Elementary-school students may see a fraction as a pair of whole numbers. An intuitive basis for developing the concept of fractional number is provided by partitioning. [4] An intuitive basis for developing the concept of fractional number is provided by seeing fractions as multiples of basic units -- for example,  $\frac{3}{4}$  is  $\frac{1}{4}$  and  $\frac{1}{4}$  and  $\frac{1}{4}$  rather than 3 of 4 parts. [5]

Elementary- and middle-school students make several errors when they operate on decimals and fractions. [6] These errors are due in part to the fact that students lack essential concepts about decimals and fractions and have memorized procedures that they apply incorrectly. Interventions to improve concept knowledge can lead to increased ability by 5th-graders to add and subtract decimals correctly. [7]

Students of all ages misunderstand multiplication and division. [8] Commonly held misconceptions include "multiplication always makes larger," "division always makes smaller," "the divisor must always be smaller than the dividend." Students may correctly select multiplication as the operation needed to calculate the cost of gasoline when the amount and unit cost are integers, then select division for the same problem when the amount and unit cost are decimal numbers. [9]

Numerous suggestions have been made to improve student concepts of multiplication [10] Further research is needed to determine how effective suggestions to improve student concepts of multiplication will be in the classroom. [11]

Lower middle-school students may have difficulties understanding the relationship between fractions and decimal numbers. [12] Lower middle-school students may think that fractions and decimals can occur together in a single expression, like  $0.5 + 1/2$ , or they might believe that they must not change from one representation to the other (from  $1/2$  to  $0.5$  and back) within a given problem. Instruction that focuses on the meaning of fractions and decimals forms a basis on which to build a good understanding of the relationship between fractions and decimals. Instruction that merely shows how to translate between the two forms does not provide a conceptual base for understanding the relationship. [13]

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