Research on Student Learning

Students have difficulty understanding how symbols are used in algebra. [1] They are often unaware of the arbitrariness of the letters chosen to represent variables in equations. [2] Middle-school and high-school students may regard the letters as shorthand for single objects, or as specific but unknown numbers, or as generalized numbers before they understand them as representations of variables. [3] Students' lack of awareness of the arbitrariness of the letters chosen to represent variables in equations tends to persist even after instruction in algebra. [4] Student difficulty in understanding how symbols are used in algebra is evident even in college students. [5] Long-term experience (3 years) in elementary computer programming has been shown to help middle-school students overcome difficulties in understanding how symbols are used in algebra, although short-term experiences (less than 6 months) are less successful. [6]

Students of all ages often do not view the equal sign of equations as a symbol of equivalence between the left and the right side of the equation, but rather interpret it as a sign to begin calculating. [7] For example, middle-school students may not accept statements like $3x + 4 = x + 8$ as legitimate because they think the right side should indicate the answer. Introducing the equal sign from the beginning as a symbol indicating "equivalence" between arithmetic equalities can ameliorate this difficulty. [8]

Beginning algebra students use various intuitive methods for solving algebraic equations. [9] Some of these methods may help their understanding of equations and equation solving. Students who are encouraged initially to use trial-and-error substitution develop a better notion of the equivalence of the two sides of the equation and are more successful in applying more formal methods later on. [10] By contrast, students who are taught to solve equations only by formal methods may not understand what they are doing. Students who are taught to use the method of "transposing" are found to only mechanically apply the change side/change sign rule. [11]

Students of all ages can often solve algebraic equations without a deeper understanding of what a solution is. For example, middle- and high-school students do not realize that an incorrect solution, when substituted into the equation, will yield different values for the two sides of the equation. [12] More research is needed to identify how students can come to understand what a solution means and why anyone would want to find it. [13]

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